Determinants of Technical Efficiency in Oklahoma Schools: A Stochastic Frontier Analysis

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Abstract

The parameters of a stochastic production frontier and the determinants of X inefficiency are estimated simultaneously using a maximum likelihood estimator proposed by Battese and Coelli (1995). Our results indicate that additional instructional and noninstruction expenditures improve student performance, but only by a small amount. In addition, we find that school district size, teacher education and experience, and teacher salary affect the technical efficiency of schools.

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I Introduction

Knowing the education production function and understanding the sources of inefficiency in elementary and secondary education is sufficiently important to justify the continuing research on these topics. Grosskopf, Hayes, Taylor and Weber (1997) estimate that the state of Texas could reduce educational spending by about 30% and achieve the same outcomes, if its schools were operated efficiently. If this estimate applies nationwide, it implies that in the United States vast amounts of resources are wasted in education. Whatever the value of the wasted resources, it is surely small compared to the human and economic costs of inferior education.

Research on the education production function has resulted in the 'Does Money Matter?' (Burtless, ed 1996) controversy. The consensus appears to be that providing more resources–money–to schools may improve outcomes, but that there is no guarantee. Hanushek (1996), who shows that U.S. schools have had large increases in resources with little if any improvement in outcomes, argues that we do not know how to improve school systems' efficiency, and that the failure to observe improved performance along with the increased resources demonstrates inefficiency. Hedges and Greenwald (1996), among others, argue that the absence of improved performance associated with the increased resources is because of confounding factors that are not held constant in this simple comparison. Hanushek, Hedges and Greenwald, and others agree that schools and teachers matter, but Hanushek, particularly, believes that such indicators as education and experience are not reliable measures of quality in schools and teachers. No easy decision rules exist for local or state decision makers to judge whether schools are receiving an appropriate amount of resources and whether the resources received are being used effectively.

Hanushek's position is much like that of Chubb and Moe (1990), who find that student ability, school organization, and family background are the chief determinants of student success. They argue that school autonomy is a crucial element of effective school organization. They present evidence that schools where principals and teachers have more autonomy are more effective. Moreover, their regression analysis suggests that a school's economic resources are not a determinant of student achievement. This contradicts their descriptive analysis, which shows that high-performance schools use significantly more resources. They conclude that the simple correlation between student performance and school resources disappears when other variables, presumably family background, are included in the regression. The relationship between resources and performance, however, might be hidden by technical inefficiency.

To the extent that the conclusions about the effects of resources and teacher quality depend upon the traditional estimate of education production functions, these conclusions are suspect. The presence of X inefficiency (Levin 1997) in schools may cause serious bias in the estimated parameters. Recent econometric estimates of models that account for X inefficiency show a positive marginal effect of resources on performance (Bates 1997), (Brewer 1996), (Deller and Rudnicki 1993), and (Ruggiero 1996).

In this paper we estimate a stochastic production frontier, which like other such estimates, finds that economic resources matter for performance. More important than this finding, however, is our use of the estimator proposed by Battese and Coelli (1995) to simultaneously estimate the parameters of the stochastic frontier and the determinants of X inefficiency. In addition, the maximum likelihood estimator also permits the prediction of the inefficiencies for each individual for each time period that data are available. To our knowledge, this is the first application of this technique to the 'money matters' controversy, and, consequently it yields new information about the sources of school efficiency. In particular, we obtain estimates of the effects of school district size, teacher education and experience, and teacher salary that may be helpful in the policy conversation.

II The Stochastic Frontier Model

A number of studies have estimated stochastic production frontiers and used the predicted efficiencies in a second stage regression to determine reasons for differing efficiencies. In the first stage the predicted inefficiencies are estimated under the assumption that they are independently and identically distributed. Regressing other variables on the inefficiencies in a second stage is a clear violation of the independence assumption. Kumbhakar, Ghosh and McGuckin (1991) find at least two serious problems with such a procedure. First, technical inefficiency may be correlated with the inputs; if so the inefficiencies and the parameters of the second stage regression are inconsistently estimated. Second, the use of OLS in the second stage ignores the fact that the dependent variable (technical inefficiency) is inherently one-sided. OLS may yield predictions that are inconsistent with this fact and it is therefore not appropriate.[1]

Kumbhakar et al. (1991), Reifschneider and Stevenson (1991), and Huang and Liu (1994) have proposed models of technical inefficiency in the context of stochastic frontier models. In these cross-sectional models, the parameters of the stochastic frontier and the determinants of inefficiency are estimated simultaneously given appropriate distributional assumptions about the model's errors.

Battese and Coelli (1995) proposed a stochastic frontier model for use with panel data in which the inefficiencies can be expressed as specific functions of explanatory variables. The model can be expressed as

$$Y_{it} = x_{it}\beta + (V_{it} - U_{it}) \qquad i = 1, \ldots, N \quad t = 1, \ldots, T \qquad (1)$$

where Y_{it} is the production of firm i in time period t; x_{it} is a $kx1$ vector of inputs; β is a vector of unknown parameters; V_{it} are random variables which are assumed to be independently and identically distributed $N(0, \sigma_v^2)$ and independent of U_{it} which are non-negative random variables that account for technical inefficiencies in production; U_{it} are assumed to be independently distributed as truncations at zero of the $N(m_{it}, \sigma_U^2)$ distribution. The mean inefficiency is a deterministic function of p explanatory variables:

$$m_{it} = z_{it}\delta \tag{2}$$

where δ is a $px1$ vector of parameters to be estimated. Following Battese and Corra (1977) let $\sigma^2 = \sigma_V^2 + \sigma_U^2$ and $\gamma = \sigma_U^2/(\sigma_V^2 + \sigma_U^2)$.

The inefficiencies, U_{it}, in equation (1) can be specified as:

$$U_{it} = z_{it}\delta + W_{it} \tag{3}$$

where W_{it} is defined by the truncation of the normal distribution with mean zero and variance, σ^2. Then, the technical inefficiency of the ith firm at time t is

$$\mathrm{TE}_{it} = \exp\left(-U_{it}\right) = \exp\left(-z_{it}\delta - W_{it}\right) \tag{4}$$

The conditional expectation of TE_{it} is given in equation (A.10) of Battese and Coelli (1993) which can be used to produce predictions for each school for each time period.

The parameters of the model (β, δ, σ^2, and γ) are estimated using the maximum likelihood estimator (MLE); the log-likelihood function can be found in the appendix.[2]

Translog Production

The translog functional form is used because it offers great flexibility in specifying the nature of production. The translog model can be interpreted as a second-order approximation to the unknown, but true, functional form.

In this paper output is measured as average district performance on one of several standardized tests. The inputs are functions of instructional expenditures per student ($I\$/S$) and other expenditures per student ($O\$/S$). For Oklahoma school production the basic translog model is:

$$\ln Score_{it} = \beta_0 + \ln(\frac{I\$}{S})\beta_1 + \ln(\frac{O\$}{S})\beta_2 + [\ln(\frac{I\$}{S})]^2\beta_3 + \\ [\ln(\frac{O\$}{S})]^2\beta_4 + \ln(\frac{I\$}{S})\ln(\frac{O\$}{S})\beta_5 + (V_{it} - U_{it}) \quad (5)$$

Equation (3), however, ignores the role of student characteristics and family background in educational achievement. Data available to measure these effects are percent of students eligible for subsidized lunch (LUNCH) as a measure of poverty, percent minority students (MIN), and percent of students classified as possessing limited English proficiency (LEP). We rewrite the production frontier as

$$\ln Score_{it} = \beta_0 + \text{LUNCH}\beta_1 + \text{MIN}\beta_2 + \text{LEP}\beta_3 + ln(\frac{I\$}{S})\beta_4 + \ln(\frac{O\$}{S})\beta_5 \\ + [\ln(\frac{I\$}{S})]^2\beta_6 + [\ln(\frac{O\$}{S})]^2\beta_7 + \ln(\frac{I\$}{S})\ln(\frac{O\$}{S})\beta_8 + (V_{it} - U_{it}) \quad (6)$$

This specification follows Bradford, Malt and Oates (1969) and Ruggiero (1996) and assumes that a change in an environmental variable, e.g. LUNCH, results in a parallel shift in the frontier. Historically, many studies have shown that disadvantaged students tend to perform below average, and consequently we anticipate that LUNCH, LEP and MIN will shift the frontier down. The coefficients of the environmental variables are part of the education production function, just as are the coefficients of the spending variables. The difference is that the district administrators, within limits, allocate resources between teachers and other inputs, but they have no choice over the environmental variables.

Modelling Inefficiency

The inefficiencies are modeled as functions of other exogenous variables. These variables are observed factors that may explain differences in technical efficiency across school districts in Oklahoma.

The factors affecting the technical efficiency of a school district are of two types. First, are data on 3 factors that represent differences in input quality under the control of the school district: average teacher salary (SALARY), average years of experience for teachers (YRSEXP), and the proportion of teachers having an advanced degree (DEG). Second are data on quantity adjustments available to district administrators (the Student-Teacher ratio), and on adjustments available to district or state policy makers (total Enrollment and total Enrollment^2).

Economic theory predicts that the use of higher quality inputs increases output, *ceteris paribus.* Hence, a teacher's possession of an advanced degree and additional experience, to the extent that these signal higher quality, are expected to reduce inefficiency. Although Hanushek (1996) is skeptical of the usefulness of these variables, we believe that the hypotheses should be retested using the frontier estimator.

The effect of salary is more complicated. If pay were related to performance, one might argue that higher salaries would motivate, as well as result from, better teaching, just the result reported by Cooper and Cohn (1997) in their study of South Carolina's incentive pay system. Although teacher pay is usually not based on teaching performance, it is possible that higher salary may induce better performance because of its effect on morale. We believe, however, that salary reflects teacher quality because of selectivity. The higher the salaries offered in the district, the better the applicant pool from which principals or other administrators choose. The better the applicant pool, the more likely is an administrator to make a good draw. Furthermore, a higher salary slows the attrition of teachers with higher opportunity cost, presumably the more productive teachers.

There is accumulating evidence that smaller classes yield better education outcomes (Krueger 1997). However, smaller classes are also more costly for the district to provide. Hence, smaller classes may actually be less efficient than larger ones even though they generate higher test scores.[3]

Finally, we believe that the size of a school district may affect its organizational efficiency. Chubb and Moe (1990), for instance, find a modest positive effect of school size on organizational efficiency.

III Data

In early 1990 the Oklahoma legislature passed a comprehensive educational reform bill with the hope of improving educational performance in K-12 public schooling. One of the bill's provisions sets up the Education Indicators program whose express purpose is to assess the performance of public schools and districts. The data used in this study were obtained from reports compiled by the Oklahoma Office of Accountability under the Education Indicators program. The data are from the academic years 1990-1991 through 1994-1995 and include 418 school districts in the state of Oklahoma.[4] The output variables include results in percentiles of the Iowa Test of Basic Skills (ITBS) for grades 3 (IT3) and 7 (IT7), and of the Test of Achievement and Proficiency (TAP) for grades 9 (TAP9) and 11 (TAP11). District enrollment (ADM) is measured as the average daily membership in the school district for the year rounded to the nearest whole number. It is calculated by dividing the total days of membership throughout the year by the number of days taught. Instructional expenditures per student ($I\$/S$) is the total expenditures for the school district devoted to instruction divided by ADM. Other expenditures ($O\$/S$) is the expenditure per student devoted to administrative and other school operations. Salary (SALARY) is the average teacher salary computed by dividing the gross salaries and fringe benefits of the district by the number of full-time equivalent (FTE) teachers for the school year. Each of the variables measured in dollars has been deflated by the consumer price index.

The Oklahoma data are particularly useful in estimating educational production functions because of the large variation in many of the variables. Scanning the ranges or observing the sizes of the standard deviation relative to the mean in Table 1 shows that variation. The variation in spending across districts is particularly relevant because it helps establish the exogeneity of spending per student. An important source of this variation is created by the way local property taxes flow to the school district. In particular, a significant portion of local property taxes flow from public utilities – electric power generation, natural gas service, and pipelines – which are taxed based on the location of their physical plant. This means that a school district fortunate enough to have a power generating plant (or other utility) in its confines has a high revenue base independent of the income of district residents. The exogenous variation in revenue and spending per student helps to identify the production function. Furthermore, nearly all Oklahoma school districts tax local property at the maximum rate allowed by the state constitution. Therefore, local variations in demand have no straightforward way of being translated into variations in expenditure.[5]

Years of experience is measured by dividing total years experience in the district by the number of FTE teachers. The percentage of teaching staff with an advanced degree is computed by dividing the number of FTE teachers with a master's degree or higher by the total number of FTE teachers. LUNCH is the percentage of Oklahoma students eligible for federally funded or reduced payment lunch in the school. MIN is the percent of "nonwhite" students (e.g., American Indian, black, Hispanic, Asian) in the district. The data also include the percent of students classified as possessing limited English proficiency (LEP). Summary statistics of the variables can be found in Table 1.

IV Results

The MLEs are obtained using each of 4 output measures: 3rd grade, 7th grade, 9th grade and 11th grade test scores. The coefficient estimates and their corresponding t-ratios for the frontier production functions are presented in Table 2. Output elasticities have been computed based on the mean values of $\ln(I\$/S)$ and $\ln(O\$/S)$ and are summarized in Table 3.

The elasticities of test scores with respect to instructional and noninstructional spending are statistically positive at the 5% level. An increase in instructional expenditures per student is estimated to increase test scores for all grades. The effects are generally larger in grade 3 than in the other grades considered. Even in grade 3 the effect of increasing the instructional spending per student is small. A one percent higher level of instructional spending is predicted to increase third grade test scores by 0.29 percent and the other test scores by from 0.16 to 0.23 percent. Other expenditures per student also have positive elasticities with respect to test scores, ranging from 0.07 to 0.10. Although these elasticities may seem small, the instruction elasticities and the average of the instructional and noninstructional elasticities are larger than those summarized by Betts (1996). In addition, a simple calculation suggests underspending on instruction. At the means, a one percent decrease in noninstructional spending releases sufficient funds for a 0.6 percent increase in instructional spending. The reduction in noninstructional spending would reduce third grade test scores by 0.08 percent, but the increase in instructional spending would increase them by 0.17 percent. Smaller net improvements would be realized with this same reallocation for grades 9 and 11, but it would cause a slight test score reduction for 7th grade. Although the exercise must be interpreted with care, it suggests the kinds of analyses possible with a well-estimated stochastic frontier. The small elasticities (which are evaluated at the sample means) indicate that improving test scores through additional spending is possible at the margin, but substantial improvements in district performance are probably prohibitively expensive.

As expected, LUNCH and MIN each shift the frontier down for all grades considered. These results simply confirm the conventional wisdom regarding the importance of variation in students, their family backgrounds, income and so on for educational performance. LEP, however, is not significant. If school districts could operate on the frontier, the estimate shows that spending more per student in districts with higher percentages of disadvantaged students could overcome some of the impediments to learning created by the environment.

According to our estimates increasing teacher salary reduces inefficiency by statistically significant amounts in all grades considered. Likewise, increasing the percentage of faculty with advanced degrees improves technical efficiency in all grades. Years experience increases technical efficiency in all grades, except for grade 3. (It is not implausible that youthful exuberance could over come lack of experience in early but not in later grades, but we do not want to push the point.) Although finding one of these quality indicators significant in a study is not unusual, finding all three indicators significant is. We believe that this is a result of the efficiency gains obtained by simultaneously estimating the parameters of the stochastic frontier and the determinants of inefficiency using the MLE.

Larger student/teacher ratios reduce technical inefficiency in grades 3, 7 and 9. This result does not contradict the estimated positive elasticity for instructional spending. It simply suggests that some types of instructional spending have bigger returns than others. In particular, the results suggest that using higher salaries to attract more-qualified and more-experienced teachers is an effective way to improve efficiency. Reducing class size from its existing levels is not.

Another policy recommendation comes from the estimate of the effects of size on efficiency. According to our estimates, larger districts have a greater degree of technical efficiency. Although the inefficiency decreases at a diminishing rate, the optimal size for technical efficiency is in the range of 18,000 to 22,000 students. Only the Oklahoma City and Tulsa City school districts in Oklahoma have enrollments beyond this range (over 35,000 were enrolled in each district). Thus, these results suggest that increasing school district size will improve efficiency.

To our knowledge, the results are unique in showing relationships between inefficiency and other variables. Bates (1997) and Deller and Rudnicki (1993) test for, but do not find, such relationships. Bates uses simple correlation to test for relationships between efficiency and teaching expenditure, and nonteaching expenditure, and socioeconomic background; the correlations are only 0.04, -0.09, and 0.04. Deller and Rudnicki use several parametric and nonparametric tests for an association between efficiency and school administrative type, school administrative spending, and school size. They find no patterns. Our detection of a relationship may be due to the use of a more efficient one-stage estimator for the frontier and the determinants of inefficiency.

Efficiency of High Schools

In the preceding models, no effort was made to control for the input quality beyond student characteristics. For instance, if a school district performs poorly in grades K-9, even a well-run effective high school might produce students with low test scores. Consequently, another model is proposed for grade 11 where the cohort's performance in grade 9 enters the model as an explanatory variable. Hence, equation (4) includes $\ln score_{i,t-2}$ as an explanatory variable. This means that the panel now only has 3 observations for each school district.

The results are found in the last two columns of Table 2. The elasticity of the 11th grade scores with respect to the 9th grade test scores is 0.60. This elasticity is at least twice that of any of the spending elasticities previously reported. This may suggest that improving elementary performance may be the best way to improve high school performance. Although the spending elasticities in this estimate are positive, they are small, 0.04 and 0.05. Moreover, they suggest that noninstructional spending may be under emphasized, if we are concerned with the ability of high schools to add value to the students coming out of lower grades.

LUNCH is a significant environmental variable. Not only is this poverty indicator associated with a lower frontier in terms of student test scores, but it reduces the ability of high schools to overcome past poor performance. MIN is not significant in this "value-added" equation, perhaps suggesting that environment handicaps due to discrimination play their biggest role in the early years. LEP takes a positive and significant coefficient, suggesting that limited In general, English proficiency is not a problem in Oklahoma.

Of the teacher characteristics, years of experience appears to be more important than either the possession of an advanced degree or of salary. More experience is associated with greater technical efficiency in the value-added model. Although advanced degree and salary are not significant, the signs of coefficients are the same as in the earlier models. In particular salary is associated with a small increase in efficiency, with its coefficient approaching significance at 0.10. As before, larger student-teacher ratios are associated with greater efficiency, and efficiency increases at a decreasing rate with district size.

In our discussion we emphasize the results for 3rd grade and 11th grade. The third grade results are important because they measure student performance early in the students' schooling and have the closest correspondence between outputs and inputs. The third grade observations most closely meet the criteria suggested by Ferguson and Ladd (1996) for using one-year district observations for spending, school variables, and background variables: stability in these variables over time and in the allocation of inputs to schools over districts. The consistency of the grades 7, 9, 11 results with the grade 3 results may indicate that these assumptions may be appropriate for the other grades as well. Nevertheless, it is important to compare the results of this specification with the results from a value-added approach. Comparing the 11th grade results with and without 9th grade constant shows that elasticities are smaller and that the determinants of inefficiency are estimated with less precision in the value-added equation. In general, however, the results for the value-added equation are similar to those for grade 3.

The results suggest in general spending more on instruction will improve test scores. In addition, they suggest that allocating more resources to instruction (and away from noninstruction) can lead to small improvements in school district performance, particularly in lower grades.

Student characteristics appear to be important, but are largely beyond a district's control. Public schools educate (or try to, anyway) all eligible children in a district; they cannot pick and choose who to admit to their schools.[6]

Robustness

The residual differences in district performance may include factors that are beyond its control and result in performance differences that are not due to inefficiency. Including available information on student characteristics as done above is one way to control for some of these differences. A more traditional approach would be to include district level dummy variables in the production function (first stage) portion of the model. This would effectively eliminate any biases in the parameter estimates that arise from the omission of time-invariant variables (that are possibly unobservable) that are correlated with included explanatory variables. Unfortunately, this also serves to mask possible inefficiency by allowing each district to essentially have its own frontier.

To gain an idea of whether omitted explanatory variables bias the estimated elasticities, a fixed effects model is estimated for the augmented production function. The resulting elasticities and their estimated standard errors are presented in table 5. Although the elasticities are not directly comparable,[7] their similarity to those in table 3 is striking. In the Grade 3, 7, and 11 results the estimated elasticity of test scores with respect to I$/S are within .025 of one another. In the fixed effects model, O$/S elasticities tend to be smaller, but are estimated to lie within a very narrow range, i.e., between .05 and .074. Estimator precision is relatively low for Grade 9 and for Grade 11 holding 9th grade performance constant. Consequently, the elasticities of test scores with respect to I$/S are insignificant in these models. Nevertheless, the overall similarity between the two sets of results increases our confidence that biases induced by omitted variables, if any, are small.

Measuring Efficiency

The estimation of the models by maximum likelihood enables the computation of estimates of technical efficiency for each school district for each year. In Table 4 are the median predicted efficiencies for each grade and year of the study. Grade 11 tends to be the least efficient except when 9th grade achievement is held constant. Then, grade 11 is the most efficient. If prior achievement is not held constant, grade 7 is most efficient. Grade 9 reaches maximum efficiency in 1992, and grades 7-11 show continuous improvement. Again, the measured inefficiency results for the value-added equation do not differ fundamentally from the grade 3 results.

The efficiency levels in this study lie between those of Grosskopf et al. (1997) and Deller and Rudnicki (1993). For instance, our median 3rd grade efficiency is 0.90 compared to a mean value of 0.71 for the former and a median of 0.91 Although the difference between 0.91 and 0.90 is trivial, Oklahoma has a greater concentration of school districts with low efficiencies than Deller and Rudnicki found for Maine.

V Conclusion

Without school level data the results should be interpreted cautiously. Nevertheless, they suggest that although money matters, there is wide variation in the efficiency with which districts use available resources to educate students. Large districts tend to be more efficient than small ones. Teacher characteristics are important, although for the high school value added equation, experience is the only characteristic that counts. Youthful teachers may be more effective in lower grades, but more experienced ones are more effective in higher grades. Higher salaries may attract better teachers who in turn improve district efficiency despite their higher costs. There also is some evidence that school districts could benefit from reallocating money away from noninstructional purposes to instruction and from district consolidation.

Studies of education production and its efficiency suggests that: (a) schools or school districts matter (Deller and Rudnicki 1993), (b) principals matter (Chubb and Moe 1990) teachers matter (Ferguson and Ladd 1996); class size matters (Krueger 1997); computers matter (Betts 1995), and so on. In addition, our study suggests that teacher experience, advanced degrees, and salary also matter.

Notes

[1]See Kumbhakar et al. (1991) for discussion.

[2]Computations were performed using FRONTIER 4.1 (Coelli 1996).

[3]One of the limitations of our data are that student-teacher ratios are reported at the district level, and therefor may not be a very accurate measure of actual class size.

[4]There are actually over 600 school districts in Oklahoma. We eliminated from our sample all of the so-called "dependent" districts that do not offer 1st through 12th grades. In addition, in the years since 1995 changing variable definitions and test instruments have made panel studies using the Oklahoma data difficult.

[5]This is similar to the argument that the passage of California's Proposition 13 "eliminated the demand side for local education expenditures." used by

Dynarski, Schwab and Zampelli (1989).

[6]Of course, districts are able to effectively accomplish the same thing on a school by school basis by reassigning children to other schools within its district. The aggregation of our data by district and not by school does not allow us to comment on this proposition.

[7]The slope coefficients in the fixed effects model are estimated under completely different assumptions about the behavior of the random error and exclude the influence of "second stage" model.

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Appendix

Battese and Coelli (1993) develop the likelihood function in the following way.

The pdf of v_{it} is

$$f_V(v) = \frac{\exp\left(-\frac{1}{2}v^2/\sigma_V^2\right)}{\sqrt{2\pi}\sigma_V} \qquad -\infty < v < \infty \tag{7}$$

The pdf of the truncated normal density is

$$f_U(u) = \frac{\exp\left(-\frac{1}{2}(u - z\delta)^2/\sigma_U^2\right)}{\sqrt{2\pi}\sigma_U \Phi(z\delta/\sigma_U)} \qquad u \geq 0 \tag{8}$$

where the subscripts, i and t, have been omitted for convenience, and the function $\Phi()$ is the distribution function for the standard normal random variable.

Let the overall equation error of the linear model be denoted, E, and note that $V = E + U$. Given the statistical independence of V and U, the joint density of E and U is obtained by multiplication. This yields:

$$f_{E,U}(e, u) = \frac{\exp\left(-\frac{1}{2}[(e+u)^2/\sigma_V^2] + [(\mu - z\delta)^2/\sigma_U^2]\right)}{2\pi\sigma_U\sigma_V\Phi(z\delta/\sigma_U)} \qquad u \geq 0 \tag{9}$$

Using the reparameterization $\mu_* = \frac{\sigma_V^2 z\delta - \sigma^2 e}{\sigma_V^2 + \sigma_U^2}$ and $\sigma_*^2 = \sigma_U^2\sigma_V^2/(\sigma_V^2 + \sigma_U^2)$ yields

$$f_{E,U}(e, u) = \frac{\exp\left(-\frac{1}{2}[(u-\mu_*)^2/\sigma_*^2] + [(e + z\delta)^2/(\sigma_U^2 + \sigma_V^2)]\right)}{2\pi\sigma_U\sigma_V\Phi(z\delta/\sigma_U)} \qquad u \geq 0 \tag{10}$$

The marginal density of E is then obtained by integrating U out of the joint density. This yields:

$$f_E(e) = \frac{\exp\left(-\frac{1}{2}[(e+z\delta)^2/(\sigma_U^2+\sigma_V^2)]\right)}{\sqrt{2\pi}(\sigma_U+\sigma_V)^{\frac{1}{2}}[\Phi(z\delta/\sigma_U)/\Phi(u_*/\sigma_*)]} \qquad u \geq 0 \tag{11}$$

The density function for production, Y_{it}, is then

$$f_{Y_{it}}(y_{it}) = \frac{\exp\left(-\frac{1}{2}\left\{\frac{(y_{it}-x_{it}\beta+z_{it}\delta)^2}{\sigma_V^2+\sigma_U^2}\right\}\right)}{\sqrt{2\pi}(\sigma_U+\sigma_V)^{\frac{1}{2}}[\Phi(d_{it})/\Phi(d_{it}^*)]} \tag{12}$$

where $d_{it} = z_{it}\delta/\sigma_U$, $d_{it}^* = u_{it}^*/\sigma_*$, and $u_{it}^* = [\sigma_V^2 z_{it}\delta - \sigma_U^2(y_{it} - x_{it}\beta)]/(\sigma_V^2+\sigma_U^2)$.

Defining $\sigma_S^2 \equiv \sigma_V^2 + \sigma_U^2$ and $\gamma \equiv \sigma_U^2/\sigma_S^2$, it follows that the log-likelihood is

$$\begin{aligned} L(\beta,\delta,\gamma,\sigma_S^2) = & -\frac{1}{2}\sum_{i=1}^{N} T_i\{\ln 2\pi + \ln\sigma_S^2\} \\ & -\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T_i}\{(y_{it}-x_{it}\beta+z_{it}\delta)^2\sigma_S^2\} \\ & -\sum_{i=1}^{N}\sum_{t=1}^{T_i}\{\ln\Phi(d_{it}) - \ln\Phi(d_{it}^*)\} \end{aligned} \tag{13}$$

where $d_{it} = z_{it}\delta/(\gamma\sigma_S^2)^{\frac{1}{2}}$, $d_{it}^* = \mu_{it}^*/[\gamma(1-\gamma)\sigma_S^2]^{\frac{1}{2}}$, $\mu_{it}^* = (1-\gamma)z_{it}\delta - \gamma(y_{it}-x_{it}\beta)$, and $\sigma_* = [\gamma(1-\gamma)\sigma_S^2]^{\frac{1}{2}}$.

For additional details, see Battese and Coelli (1993)

Variable	T	Mean	Std Dev	Minimum	Maximum
IT3	2073	58.0424	11.2374	7.0000	94.00
IT7	2072	58.0275	9.3049	24.0000	88.00
TAP9	2075	58.0679	9.3618	18.0000	87.00
TAP11	2061	52.9184	9.9580	13.0000	83.00
Total$/S	2090	3648.11	814.6493	2160.00	9847.00
I$/S	2090	2275.48	480.3793	1427.00	5593.00
O$/S	2090	1372.63	425.4767	573.000	5598.00
ADM	2090	1338.65	3445.00	86.000	41831.00
TEACHERS	2090	84.4167	199.3964	10.000	2553.00
SALARY	2090	25753.31	2533.92	19155.0	33474.00
DEGREE	2090	35.7406	14.1658	2.0000	89.00
YRSEXP	2090	12.0636	2.5242	5.0000	27.00
LUNCH	2090	46.5972	17.7563	5.0000	100.00
MIN	2090	23.6045	16.8830	0	100.00
LEP	2090	12.0050	3.5504	0	39.00
STU/TEACH	2090	14.0523	2.3356	6.4166	24.43

Table 1: Means, Standard Deviations, Minimum and Maximum values of each of the variables

	Grade 3		**Grade 7**		**Grade 9**		**Grade 11**		**Grade 11**	
	Coef	Tval	Coef	Tval	Coef	Tval	Coef	Tval	Coef	Tval
Constant	-23.4839	-15.76	-0.142	-0.12	2.017	0.86	-5.499	-3.41	5.386	2.355
$\ln(\text{9th G. Score})_{t-2}$									0.601	25.232
School Lunch	-0.0021	-7.34	-0.003	-10.99	-0.003	-15.16	-0.004	-15.76	-0.002	-6.407
Minority	-0.0012	-4.49	-0.002	-8.36	-0.002	-8.01	-0.001	-5.91	2.42E-04	0.958
LEP	0.0011	0.95	0.001	0.93	-0.001	-0.94	-3.27E-04	-0.33	0.002	2.185
$\ln(I\$/S)$	6.2109	8.47	-0.583	-1.04	-0.768	-0.97	0.898	1.25	-1.352	-1.548
$\ln(O\$/S)$	1.0680	1.51	1.664	2.99	1.239	2.20	1.599	2.26	0.286	0.388
$\ln^2(I\$/S)$	-0.3438	-3.38	0.079	1.08	0.126	1.50	-0.080	-0.82	-0.094	-0.936
$\ln^2(O\$/S)$	-0.0040	-0.12	-0.081	-2.92	-0.014	-0.45	-0.152	-3.93	-0.236	-5.469
$\ln(I\$/S)\ln(O\$/S)$	-0.1268	-0.99	-0.062	-0.67	-0.133	-1.38	0.074	0.60	0.405	3.391
Constant	2.6665	7.69	2.046	8.47	1.351	5.03	1.674	6.82	1.058	2.112
Salary	-0.0002	-7.69	-0.00011	-7.65	-4.87E-05	-2.92	-0.00010	-5.59	-2.72E-05	-1.241
Adv. Degree	-0.0071	-5.24	-0.003	-3.36	-0.005	-4.91	-0.001	-0.82	-0.001	-0.985
Yrs. Experience	0.0298	3.98	-0.010	-2.13	-0.035	-3.97	-0.017	-3.06	-0.068	-5.562
Enrollment	-0.2163	-6.23	-0.147	-4.14	-0.187	-25.68	-0.197	-19.25	-0.130	-8.488
Enrollment^2	0.0052	6.19	0.004	4.33	0.005	18.85	0.005	12.82	0.003	4.872
Student/Teacher	-4.214	-3.10	-4.584	-2.63	-35.460	-2.64	0.371	0.37	-32.700	-2.370
σ^2	0.2084	10.94	0.079	9.16	0.124	6.56	0.119	8.08	0.102	5.269
γ	0.9380	118.10	0.881	53.71	0.935	101.42	0.919	72.09	0.918	46.024

Table 2: Maximum likelihood estimates of the production function and its stochastic frontier

	Grade 3	Grade 7	Grade 9	Grade 11	Grade 11 W/ 9th g. score
$(I\$/S)$	0.2854	0.1570	0.1864	0.2270	0.0397
Standard Error	0.0276	0.0223	0.0239	0.0254	0.0281
$(O\$/S)$	0.0801	0.1031	0.0785	0.0728	0.0484
Standard Error	0.0193	0.0157	0.0155	0.0167	0.0201

Table 3: Elasticities of test scores with respect to instructional and noninstructional expenditures per student evaluated at their means

			Year		
	1990	1991	1992	1993	1994
Grade 3	0.865	0.874	0.886	0.901	0.910
Grade 7	0.906	0.905	0.925	0.927	0.928
Grade 9	0.909	0.917	0.925	0.917	0.919
Grade 11	0.874	0.882	0.905	0.903	0.911
Grade 11 w/ 9th const			0.938	0.934	0.933

Table 4: Predicted median efficiencies by year for grades 3, 7, 9, and 11 for model 1 and for grade 11 when that cohort's 9th grade score is included in the production function.

	Grade 3	Grade 7	Grade 9	Grade 11	Grade 11 W/ 9th g. score
$(I\$/S)$	0.3048	0.1349	0.0290	0.2185	-0.0419
Standard Error	0.0370	0.0278	0.0287	0.0343	0.0482
$(O\$/S)$	0.0739	0.0599	0.0494	0.0478	0.0510
Standard Error	0.0277	0.0208	0.0214	0.0250	0.0331

Table 5: Elasticities of test scores with respect to instructional and noninstructional expenditures per student evaluated at their means estimated using a fixed effects model